Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(NEW) EXAMINATION – WINTER 2022

Subject Code:3140708 Date:16-12-2022

Subject Name:Discrete Mathematics

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	A committee 5 persons, is to be formed from 6 men and 4 women. In how many ways this can be done when (i) at least 2 women are included, (ii) at most 2 women are included.	**03**
	(b)	If $A=\{4,5,7,8,10\}$, $B=\{4,5,9\}$ and $C=\{1,4,6,9\}$, then verify that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.	**04**
	(c)	Define Functionally complete set of connectives, Principal Disjunctive Normal Form (PDNF). Obtain PDNF for the expression $\neg((p\wedge q)\vee(\neg p\wedge q)\vee(q\wedge r))$	**07**
Q.2	**(a)**	Define Partial Order Relation. Illustrate with an example.	**03**
	(b)	Define one – one function. Show that the function $f:R\to R$, $f(x)=3x-7$ is one – one and onto both. Also find its inverse.	**04**
	(c)	Solve the recurrence relation $a_{n+2}-5a_{n+1}+6a_n=2$ with initial condition $a_0=1$ and $a_1=-1$ using method of undetermined coefficients.	**07**
		OR	
	(c)	Use generating function to solve a recurrence relation $a_n=3a_{n-1}+2$ with $a_0=1$.	**07**
Q.3	**(a)**	Define Partition of a Set. Let $A=\{1,2,3,4,5\}$ and $R=\{(1,2),(1,1),(2,1),(2,2),(3,3),(4,4),(5,5)\}$ be an equivalence relation on A. Determine the partition for R^{-1}, if it an equivalence relation.	**03**
	(b)	Draw Hasse Diagram for the lattice (S_{30},D) where S_{30} is the set of divisors of 30 and D is the relation divides.	**04**
	(c)	Show that the set S of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ where $a,b\in R$ is a field with respect to matrix addition and matrix multiplication.	**07**
		OR	
Q.3	**(a)**	Define Semi Group, Monoid. Give an example of an algebraic structure which is semi group but not monoid.	**03**

(b) Consider the a relation R defined on $A=\{1,2,3\}$ whose matrix representation is given below. Determine its inverse R^{-1} and complement R'. **04**

$$M_R=\begin{bmatrix}1 & 0 & 0\\ 1 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}$$

(c) Define free variable and bound variable. **07**
Rewrite the following argument using quantifiers, variables and predicate symbols. Prove the validity of the argument.

"All healthy people eat an apple a day.
Ram does not eat apple a day.
Ram is not a healthy person."

Q.4 (a) Define Abelian group and prove that the set $\{0,1,2,3,4\}$ is a finite abelian group under addition modulo 5 as binary operation. **03**

(b) Define even permutation. Show that the permutation $\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 5 & 6 & 2 & 4 & 1 & 3\end{pmatrix}$ is odd and $\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 6 & 3 & 4 & 5 & 2 & 1\end{pmatrix}$ is even. **04**

(c) Define Principal Ideal. Find all the principal Ideal of the ring $\left[\{0,1,2,3,4,5\},+_6,\times_6\right]$. **07**

OR

Q.4 (a) Define cut vertex. List out all the cut vertices of the graph given in Figure 1. **03**

(b) Define adjacency matrix and path matrix of a graph. Find out adjacency matrix for the graph given in Figure 1. **04**

(c) Define the terms: Simple Graph, Multi – Graph, Weighted Graph, Degree of a vertex, in degree and out degree of a vertex. Illustrate each with an example. **07**

Q.5 (a) State Lagrange's theorem and find out all possible subgroups of group $\left[\{1,-1,i,-i\},\times\right]$. **03**

(b) Define Eulerian graph, Planar Graph. **04**
Justify whether the graph given in Figure 1 is Planer or not using Euler's formula.

(c) Define Binary tree, Spanning tree, Minimal Spanning tree, Find the minimal spanning tree for the weighted graph given in Figure 2. **07**

OR

Q.5 (a) Define Cyclic group, Normal subgroup. Illustrate with an example. **03**

(b) Form a binary search tree for the data 16,24,7,5,8,20,40,3. **04**

(c) Explain Post order traversal. Given the postorder and inorder traversal of a binary tree, draw the unique binary tree. **07**
Postorder: d e c f b h i g a
Inorder: d c e b f a h g i.

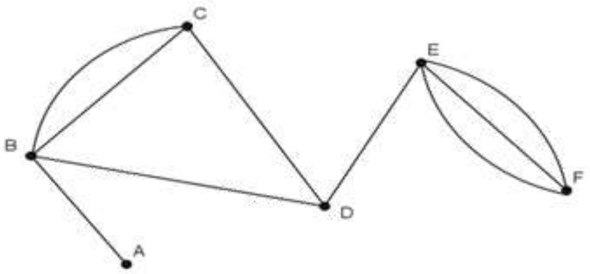

Figure 1

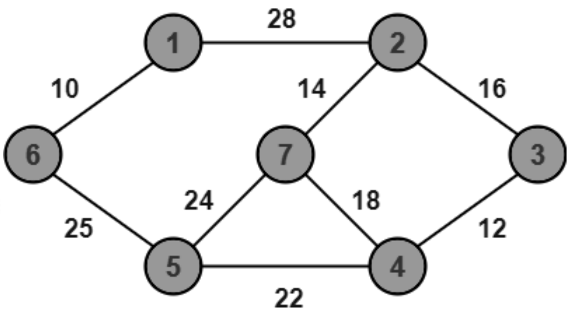

Figure 2

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